Lab 4 - AC Circuits and AC Power

Objectives

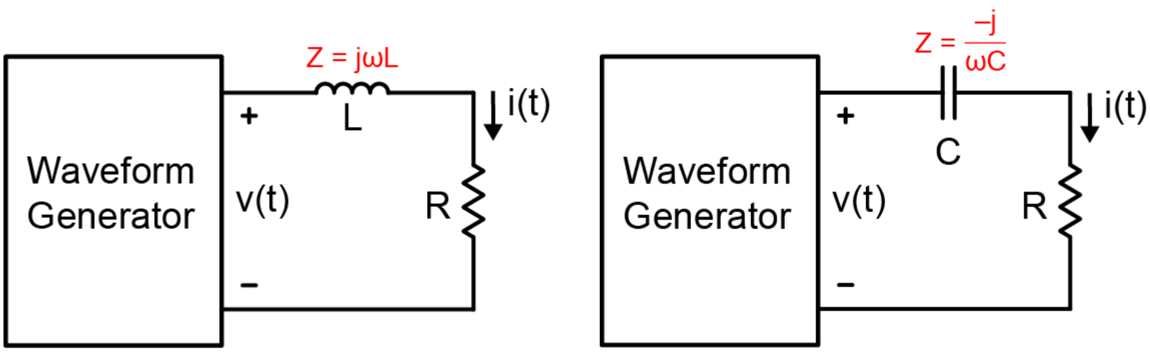
In this lab you will optimize power transfer between a source and the load in a number of different circuits. You will measure power factors and attempt to compensate circuits to achieve unity power factors.

Laboratory Equipment

In this lab you will use a function / waveform generator, digital oscilloscope, and resistors, capacitors and inductors.

Background

The first circuits that we will analyze are the simple RL series and RC series combinations: shown in below:



To these circuits we will apply sinusoidal, square, and triangular voltage waveforms. For all three you will measure both the voltage v(t) and the current i(t) and later find the power dissipated in the resistor. You can get the current through those elements by measuring the resistor voltage and dividing by the resistance. When the waveform generator is set to produce a sinusoidal voltage, you will also compute the power factor for the RL or RC combination.

You must be careful when selecting a voltage level for the test. What is the maximum voltage that the function generator can have, for example, assuming that we use a ¼ watt 51Ω resistor? From Ohms law we can calculate that the maximum allowed current is about 70 mA (average) which translates to a peak voltage of about 5 volts. The actual maximum voltage allowed on the function generator depends on the frequency. For example, for the RL circuit shown above, the voltage across the resistor can be calculated using voltage divider equations:

$$\hat{V}_R = \frac{R}{R + j\omega L}\hat{V}$$

And therefore the amplitude of the applied voltage is larger than the resistor voltage amplitude by a factor of:

$$\frac{|\hat{V}|}{|\widehat{V_R}|} = \sqrt{1 + (\omega L/R)^2}$$

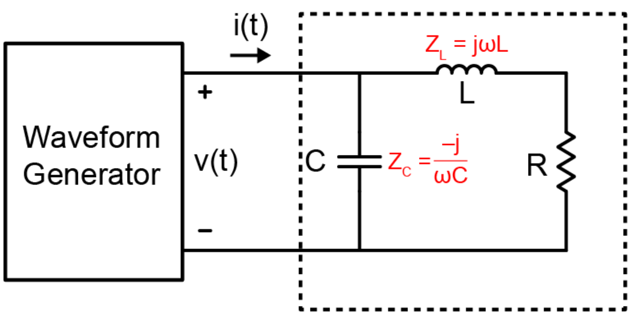
The previous factor arises from the fact that the phasor current for the RL branch is

$$\hat{I} = \hat{V}/(R + j\omega L)$$

From this formula and the definition of the power factor we can find

$$cos\,(\varphi) = \left[1 + \left(\frac{\omega L}{R}\right)^2\right]^{-1/2}$$

To adjust the power factor to one, we can add a parallel capacitor as shown below:



The total impedance of the R, L, C combination shown above is given by:

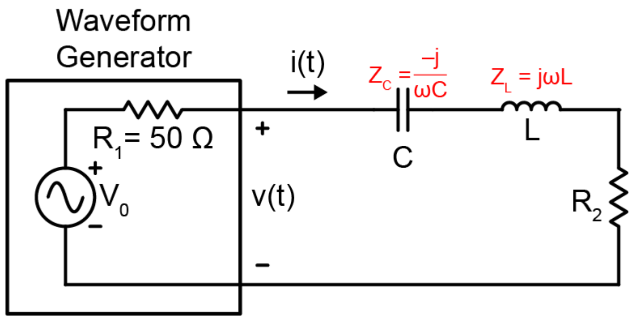
$$Z = Z_C || \,(R + j\omega L) = \left[j\omega C + \frac{1}{R + j\omega L}\right]^{-1}$$

And from this, the phasor current is found to be:

$$\hat{I} = \hat{V}/Z = \left[\frac{R}{R^2 + (\omega L)^2} + j\omega\left(C - \frac{L}{R^2 + (\omega L)^2}\right)\right]\hat{V}$$

Proper choice of the capacitance can reduce the reactive term in this expression to zero, resulting in a unity power factor. Note that the resistor could also be adjusted to cancel the reactive term for a fixed capacitance and inductance, within certain limitations. In the lab, you will show that by proper choice of the capacitor you can make the power factor equal zero.

Now consider the RLC series circuit shown here:



In order to completely describe this circuit, we must take into account that the waveform generator has an internal 50 Ω output resistance (denoted R1), which cannot be ignored. The phasor current is

$$\hat{I} = \hat{V}/\{R_1 + R_2 + j[\omega L - 1/(\omega C)]\}$$

For a given frequency ω, either the capacitance or inductance can be adjusted to achieve unity power factor. Another way of looking at this is that for a given C and L, there is one frequency where the power factor is unity. That frequency is called the resonant frequency and it is given by the formula

$$\omega_R = \sqrt{\frac{1}{LC}}$$

At this resonant frequency, the voltage across the inductor and capacitor are equal in magnitude but 180º out of phase. Consequently, the two voltages "cancel" each other out and the current is simply

$$\hat{I} = \hat{V}/(R_1 + R_2)$$

If we furthermore have that R1=R2, then the maximum power will be transferred from the source to the load resistor R2. Notice that the voltages across the two energy storage elements can be quite high. The magnitude of the voltage across either element is

$$\frac{|\widehat{V}|}{V_0} = \frac{\omega L}{R_1 + R_2} = \frac{\sqrt{L/C}}{R_1 + R_2}$$

For some of the component combinations in this lab, the voltage on the capacitor can be ten times higher than the source voltage.

Helpful Hints:

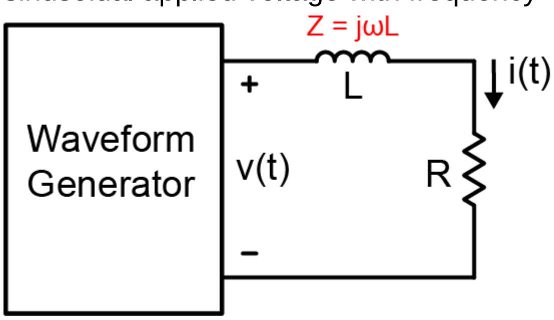
- Remember not to overheat the resistors! Calculate the maximum acceptable applied voltage and do not exceed it!
- Both the ADALM2000 and the benchtop function generators used in the laboratory have a built-in output resistance of R1=50 Ω -- you do not need to connect this resistor from your kit.

- Because the voltages in LC circuits can be much higher than the source voltage, **DO NOT TOUCH** any part of the circuit while the function generator is on! Exercise extreme caution. Make certain, as always, that the waveform generator is turned off before making any chang-es to the circuit. Make certain that the output voltage is set to the (low) value specified in the laboratory procedure.

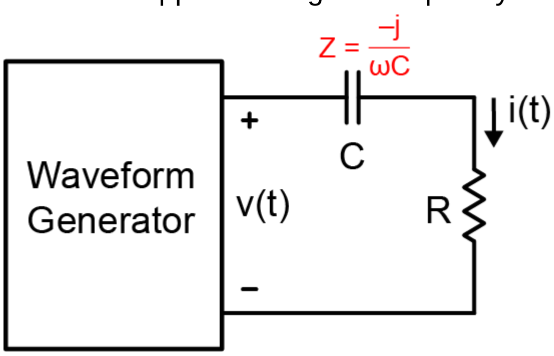
Pre-lab Preparation

Power Factor Calculations & Simulations

1. Calculate the power factor for the series RL circuit with R=200Ω and L=4.7 mH with a sinusoidal applied voltage with frequency 4 kHz:



2. Use PSpice time-domain simulation to plot the voltage across R and L of the previous step as a function of time
3. Calculate the power factor for the series RC circuit with R=200Ω and C=100 nF with a sinusoidal applied voltage of frequency 4 kHz:

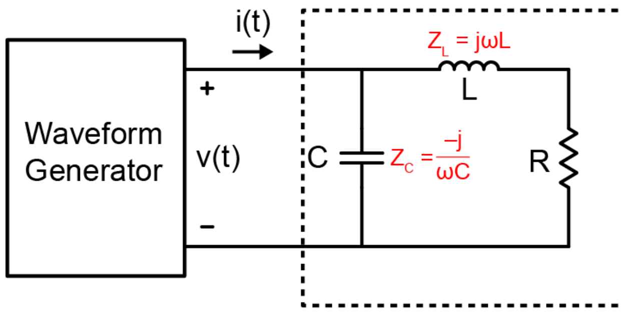


4. Use PSpice time-domain simulation to plot the voltage across R and C of the previous step as a function of time. For the PSpice simulations, use the “Transient Analysis”, and choose your simulation time window to be long enough that your answer does not depend on the initial conditions.

Power Factor Corrections

5. Compute the frequency necessary to have a power factor of one for a circuit where a 100 nF capacitor is connected in parallel with a series RL branch with R=200Ω and

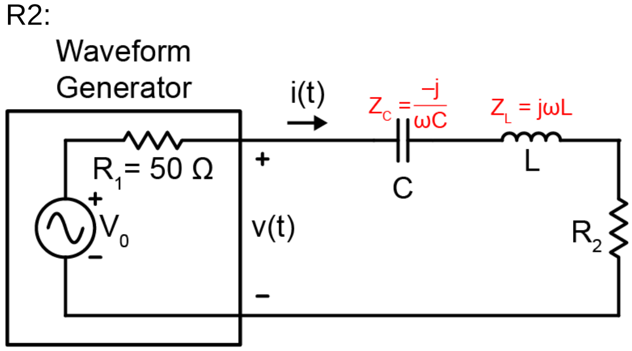
L=4.7 mH:



6. Simulate the circuit in the previous step using PSpice and plot the source current and the voltage across the resistor and the capacitor as a function of time. Again, choose a time range sufficient that the initial conditions do not matter. Compute the power dissipated in the circuit.

Resonant Circuit

7. Simulate the series RLC circuit with R1= R2=51Ω, C=100 nF and L=4.7 mH at the resonant frequency of the circuit. Plot the voltages across the inductor, capacitor, and R2:



Instructions

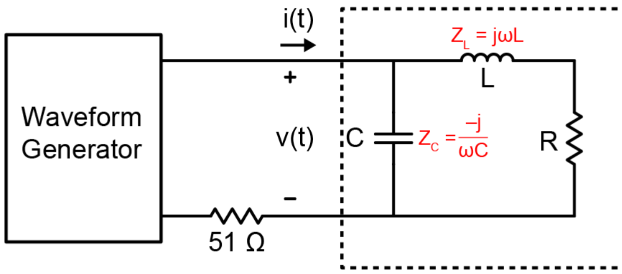
Power factor measurements

1. Construct the RL series circuit using R=200Ω and L=4.7 mH. Set the function generator to 4 kHz and use as large a voltage as possible without exceeding the ¼ W power limit for the resistor. Measure the voltages across both components for sine, square, and triangular waves.
   Because the oscilloscope probes typically have a common reference connection, you may find that it is easiest to measure the total voltage and the voltage across one of the two components, and determine the remaining component voltage by subtraction. Plot the sinusoidal voltage across the RL branch. Calculate the power factor for the RL branch from the sinusoidal plots.
2. Construct the RC series circuit using R=200Ω and C=100 nF. Set the function generator to 4 kHz and use as large a voltage as possible. As for the RL circuit, measure the voltages across both components for sine, square, and triangular waves. Plot the

sinusoidal voltage across the RC branch, and determine the power factor for the RC branch from the sinusoidal plots.

Power factor corrections

3. Next, we will try to achieve a unity power factor by adding a parallel capacitor, as considered in the pre-lab question 5. However, in order to measure the total current using an oscilloscope, we must include an additional series resistor, as shown below:



Measure the power factor for the R,L,C combination shown in the box. (As in Lab 2, you will need to measure two sinusoidal voltages, and apply Ohms law for the 51 Ω resistor to infer the current.) Adjust the frequency until the power factor is one. Record the determined frequency.

4. Double the frequency from the last part and measure the power factor. Adjust the resistance by swapping out different values until you recover a power factor of one. Plot the resistor voltage and source current. Record the value of the required resistance.

Resonant circuit

5. Construct the series RLC resonant with R2=51Ω , C=100 nF and L=4.7 mH (R1 is the internal resistance of the waveform generator). Adjust the frequency to the resonant value and record. Plot the voltages across the inductor, capacitor, and resistor (this will require two plots).

Post-lab Analysis

Generate a lab report “following” the sample report available in Appendix A. Mention any difficulties encountered during the lab. Describe any results that were unexpected and try to account for the origin of these results (i.e. explain what happened). In ADDITION, answer the following questions:

Power factor measurements

1. What is the experimental power factor for the RL circuit? Show your calculation. Does it agree with the theoretical value given the measured parameter values? Explain.
2. Does the voltage across the inductor of the RL circuit always resemble the input voltage in shape (for the sine, square wave, and triangle wave inputs)? If not, why not.
3. What is the experimental power factor for the RC circuit? Show your calculation. Does it agree with the theoretical value given the measured parameter values? Explain.

4. Does the voltage across the capacitor of the RC circuit always resemble the input voltage in shape (for the sine, square wave, and triangle wave inputs)? If not, why not.

Power factor corrections

5. For experimental step 3, did the actual frequency needed to achieve a unity power factor agree with the calculation? If not, speculate as to why not.
6. For experimental step 4, were you able to achieve a power factor of one? If so, what was the needed resistance? If not, why not?

Resonant circuit

7. For experimental step 5, did the experimental resonant frequency agree with the theoretical value? If not, why not?
8. Did the experimental ratio of the maximum inductor voltage to maximum resistor voltage agree with the simulation? Explain.